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2023 厦大杯中学物理比赛

2023 XMUM Cup Physics Competition for Secondary School

Question	Acceptable answers		Question	Acceptable answers
1	0.56		16	0.23
2	12		17	0.72
3	8.6		18	26
4	2.1		19	0.54
5	1.2		20	2.6
6	35		21	95
7	4.4		22	13
8	27		23	98
9	78		24	10
10	0.69		25	94
11	42		26	0.52,0.53
12	0.64		27	3.6
13	0.86		28	270
14	7.2		29	5.2,5.3,5.4
15	87		30	12

Topics	Questions	No. Questions
Mechanics	2,3,4,5,6,7,8,9,10,11,12,13,26,29,30	15
Thermal Physics	14,15	2
Electromagnetism	18,19,20,21	4
Electronics Circuit	22,23,24,25	4
Modern Physics	27,28	2
Waves	16,17	2
Measurement	1	1
	Total	30

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Question 1

一般上，一位宇航员每次呼吸会消耗 26 cm^3 的氧气。若呼吸的平均频率为 0.25 Hz ，估计宇航员一天消耗多少 m^3 体积的氧气。

Typically, an astronaut consumes about 26 cm^3 of oxygen with each breath. If the average rate of breathing is 0.25 Hz . Estimate the volume of oxygen consumed by an astronaut in a day **in units of m^3** .

Ans: 0.56

$$V = 26 \times 10^{-6} \times 0.25 \times 3600 \times 24 = 0.56 \text{ m}^3$$

Question 2

一辆车(1720 kg)以速率 100 kmh^{-1} 在一平坦的路上行驶。求刹车时, 车在 55 m 的距离内停下所受到平均力的大小。答案以 10^3 N 为单位。

A car (1720 kg) is moving on a flat road with a speed of 100 kmh^{-1} . When the car is braking, determine the magnitude of the average force needed to bring the car to stop within 55 m . **Give the answer in unit of 10^3 N** .

Ans: 12

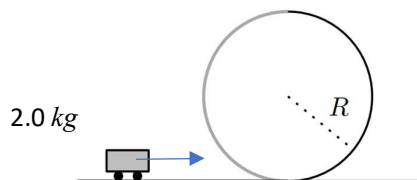
$$a = \frac{v^2 - v_0^2}{2s} = \frac{0^2 - \left(\frac{100}{3.6}\right)^2}{2(55)} = -7 \text{ ms}^{-2}$$

$$F = ma = 1720 \times 7 = 12040 \text{ N}$$

Question 3

质量为 2.0 kg 的小车进入半径为 $R = 1.5 \text{ m}$ 的环形轨道。求进入轨道时的最小速率，使小车能够完成一圈的运动。答案以 ms^{-1} 为单位。

A cart of mass 2.0 kg enters a looped track of radius $R = 1.5 \text{ m}$. Find the minimum speed upon entering the loop, such that the cart is able to make it through the full circle. **Give the answer in unit of ms^{-1}** .



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Ans: 8.6

Using the conservation of energy

$$\frac{1}{2}mu^2 = mg(2R) + \frac{1}{2}mv^2$$

And the centripetal force on the top

$$N + mg = \frac{mv^2}{R}$$

Solve u from the above two equations when $N = 0$

$$0 + mg = \frac{mu^2 - 4mgR}{R}$$
$$u = \sqrt{5gR} = \sqrt{5(9.8)(1.5)} = 8.57 \text{ ms}^{-1}$$

Question 4

A, B, C 三艘船用绳索系在一起，在冰冻的湖面上被水平外力拉动，如图 2 所示。

三艘船的质量分别为 30 kg，20 kg 和 10 kg。水平拉力为 126 N。求船 B 的加速度大小。答案以 ms^{-2} 为单位。

Three boats A, B, and C are tied and are being pulled horizontally on frictionless ice on a frozen lake using ropes as shown in Figure 2. The masses of the three boats are of 30 kg, 20 kg, and 10 kg, respectively. The pull is of magnitude 126 N. Find the magnitude of the acceleration of boat B **in unit ms^{-2}** .

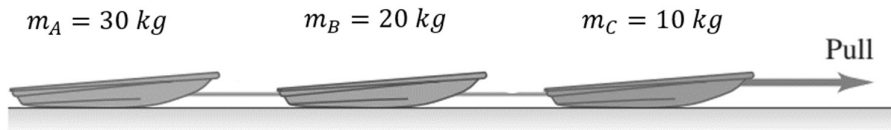


Figure 2

Ans: 2.1

$$F = (m_A + m_B + m_C)a$$
$$a = \frac{126}{30 + 20 + 10} = 2.1 \text{ ms}^{-2}$$
$$a_A = a_B = a_C = a$$

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Question 5

一个 4.70 kg 的箱子由静止从 26 m 高的建筑顶部掉下，受到的空气阻力可以忽略不计。计算箱子在撞击地面前的动能。答案以 kJ 为单位。

A 4.70-kg box is dropped from rest from the roof of a 26.0-m-tall building and feels no appreciable air resistance. Calculate the kinetic energy of the box just before it hits the ground.

Give the answer in unit kJ.

Ans: 1.2

$$\begin{aligned}W &= mgh = E_k - E_{k0} \\4.7(9.8)(26) &= E_k - 0 \\E_k &= 1197 \text{ J} = 1.2 \text{ kJ}\end{aligned}$$

Question 6

一物体在无摩擦水平面上进行简谐运动，周期为 1.20 s，振幅 0.60 m。在时间 $t = 0$ 时，物体处在平衡位置 $x = 0$ 上，正往负 x 方向移动。计算在时间 $t = 0.48$ s 时，物体离平衡位置的距离。答案以 cm 为单位。

An object is undergoing simple harmonic motion with a period 1.20 s and amplitude 0.60 m on a frictionless horizontal surface. At time $t = 0$ the object is at the equilibrium position $x = 0$, and is moving in the negative x -direction. Calculate the distance of the object from the equilibrium position when $t = 0.48$ s. **Give the answer in unit cm.**

Ans: 35

The displacement of the simple harmonic motion is described by

$$x = A \cos(\omega t + \phi_0)$$

And the velocity

$$v = -\omega A \sin(\omega t + \phi_0)$$

Given $A = 0.60 \text{ m}$, $\omega = \frac{2\pi}{1.20 \text{ s}}$

$\phi_0 = \pi/2$ because at time $t = 0$, $x = 0$ and v is negative. So,

$$x = 0.60 \cos\left(\frac{2\pi}{1.2}t + \frac{\pi}{2}\right)$$

When $t = 0.48 \text{ s}$

$$x = 0.60 \cos\left(\frac{2\pi}{1.2}0.48 + \frac{\pi}{2}\right) = -0.353 \text{ m} = -35.3 \text{ cm}$$

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Question 7

一质量可以忽略不可伸缩的细弦缠绕一质量为 M ，半径为 R 的均匀圆柱。弦的一端固定在天花板上。圆柱由静止被释放后水平下落，如图 3 所示。假设圆柱在下落过程中没有滑动发生，圆柱的转动惯量为 $I = \frac{1}{2}MR^2$ ，求圆柱的圆心下落1.5 m后，其质心的速率。答案以 ms^{-1} 为单位。

A thin inextensible string of negligible mass is wrapped around a uniform cylinder of mass M and radius R . The other end of the string is attached to the ceiling. The cylinder is released from rest and falling horizontally as shown in the figure 3. We assume no slipping occurs. The moment of inertia of the cylinder is $I = \frac{1}{2}MR^2$. When the centre of the cylinder has fallen by 1.5 m, find the speed of its centre of mass. **Give the answer in unit ms^{-1} .**

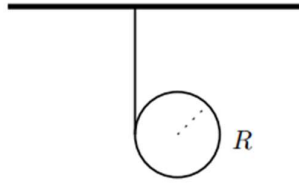


Figure 3

Ans: 4.4

By conservation of energy

$$\begin{aligned} Mgh &= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2} \\ Mgh &= \left(\frac{1}{2} + \frac{1}{4}\right)Mv^2 \\ v &= \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}(9.8)(1.5)} = 4.43 \text{ ms}^{-1} \end{aligned}$$

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Question 8

一箱子受一个与水平面成 θ 角的拉力在粗糙的平面上运动，若箱子与平面之间的滑动摩擦系数为 0.51，求角度 θ 使得维持箱子匀速运动的力最小。

A box is pulled by a force that is at an angle θ above horizontal surface. If the coefficient of kinetic friction between the object and the surface is 0.51, calculate the value of θ for which the minimum force is required to maintain constant speed.

Ans: 27

In x-direction

$$F \cos \theta - \mu N = 0$$

In y-direction

$$F \sin \theta + N - mg = 0$$

Solve θ ,

$$F \sin \theta + \frac{F}{\mu} \cos \theta - mg = 0$$
$$F = \frac{\mu mg}{\mu \sin \theta + \cos \theta}$$

Minimum force requires the denominator $\mu \sin \theta + \cos \theta$ to be maximum.

$$\tan \theta = \mu = 0.51$$

$$\theta = 27^\circ$$

Method 1. Using calculus $\frac{d}{d\theta} (\mu \sin \theta + \cos \theta) = 0 \rightarrow \tan \theta = \mu$.

Method 2. Using trigonometric relation $\cos(\theta - \phi) = \mu \sin \theta + \cos \theta$. Maximum requires $\theta - \phi = 0$. $\theta = \phi = \tan^{-1} \mu$.

Question 9

有一星球的半径是地球半径的一半，质量是两倍地球质量。取地球表面的重力加速度为 9.8 ms^{-2} ，求此星球表面的重力加速度。答案以 ms^{-2} 为单位。

A planet was found to be half the radius of the Earth, but its mass is twice the mass of the Earth. Take the gravitational acceleration on the surface of Earth 9.8 ms^{-2} , find the acceleration due to gravity on the surface of this planet. Give the answer in unit ms^{-2} .

Ans: 78

The surface gravity of a planet is $g' = \frac{GM}{R^2} = \frac{G(2M_E)}{\left(\frac{R_E}{2}\right)^2} = 8 \frac{GM_E}{R_E^2} = 8g = 78.4 \text{ ms}^{-2}$

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Question 10

三个星球的质量分别为 $M, m,$ 和 $5M$, 沿 x -轴排列成一直线。质量 M 与 $5M$ 的距离是 R , m 与 $5M$ 的距离是 x 。求比值 x/R 使得质量 m 的星球所受净重力为零。

Three planets of masses $M, m,$ and $5M$ are aligned along the x -axis. The distance between M and $5M$ is R , and the distance between m and $5M$ is x . Determine the ratio x/R such that the net gravitational force on m is zero.

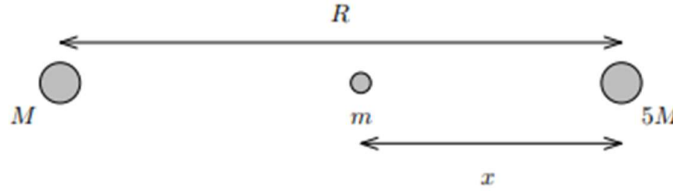


Figure 4

Ans: 0.69

$$F = -\frac{GMm}{(R-x)^2} + \frac{Gm(5M)}{x^2}$$

For $F = 0$,

$$\begin{aligned}\frac{1}{(R-x)^2} &= \frac{5}{x^2} \\ 4x^2 - 10Rx + 5R^2 &= 0 \\ x &= \frac{1}{4}(5 - \sqrt{5})R = 0.69R\end{aligned}$$

Question 11

地球同步卫星相对静止处在地球赤道上空。求卫星绕地球中心作圆周运动的轨道半径。答案以 10^6 m 为单位。

Geostationary satellite stays still relatively at the same point in the sky of the earth's equator. Determine the radius of the circular motion that the satellite must orbit. **Give the answer in unit 10^6 m .** (Mass of Earth 地球质量 $6.0 \times 10^{24} \text{ Kg}$, Universal gravitational constant 万有引力常数 $6.7 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$)

Ans: 42

$$\begin{aligned}\frac{mv^2}{r} &= \frac{GMm}{r^2} \\ v &= \frac{2\pi r}{T} \\ \frac{4\pi^2 r^2}{rT^2} &= \frac{GM}{r^2}\end{aligned}$$

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$$r = \left(\frac{GMT^2}{4\pi^2} \right)^{\frac{1}{3}}$$
$$= \left(\frac{6.7 \times 10^{-11} \cdot 6.0 \times 10^{24} (24 \times 3600)^2}{4\pi^2} \right)^{1/3} = 42 \times 10^6 \text{ m}$$

Question 12

一块 7.0 kg 的质量悬挂在弹性系数为 112 Nm^{-1} 的弹簧一端。质量被拉离平衡位置 0.60 m 后，由静止状态释放。求质量振动的频率。答案以 Hz 为单位。

A block of mass 7.0 kg is attached to a spring with Hooke's constant 112 Nm^{-1} . The block is pulled 0.60 m away from its equilibrium position and released from rest. Determine the frequency of the oscillation. Give the answer in unit Hz.

Ans: 0.64

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{112}{7}} = 0.637 \text{ Hz}$$

Question 13

两颗均匀的弹珠，质量 76 g，直径 2.0 cm，如图 5 所示堆叠在一宽度为 3.0 cm 的容器里。弹珠与容器壁的摩擦力可忽略不计。当系统平衡时，求弹珠彼此之间的作用力大小。答案以 N 为单位。

Two uniform, 76-g marbles, 2.0 cm in diameter are stacked as shown in figure 5 in a container that is 3.0 cm wide. The friction between the marbles and the wall of the container is negligible. When the system is in equilibrium, determine the magnitude of the force that each marble exerts on the other.

Give the answer in unit N.

Ans: 0.86

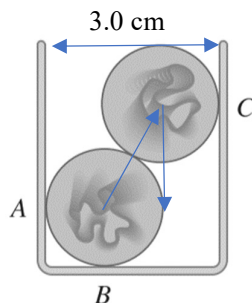


Figure 5

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$$F \cos \theta = F \frac{\sqrt{(2r)^2 - r^2}}{2r} = mg$$
$$F = \frac{2mg}{\sqrt{3}} = \frac{2 \times 0.076 \times 9.8}{\sqrt{3}} = 0.86 \text{ N}$$

Question 14

计算 12 g, -4°C 的冰完全熔化成 60°C 的水所需的最小热量。答案以 **kJ** 为单位。

Calculate the minimum required heat to convert 12 g of ice at -4°C into water at 60°C . **Give the answer in unit kJ.**

The following parameters need to be used:

Latent heat of ice(冰的熔化潜热) $l_{ice} = 3.4 \times 10^5 \text{ J/kg}$,

Specific heat of ice(冰的比热) $c_{ice} = 2.1 \times 10^3 \text{ J K}^{-1} \text{ kg}^{-1}$,

Specific heat of water(水的比热) $c_{water} = 4.2 \times 10^3 \text{ J K}^{-1} \text{ kg}^{-1}$.

Ans:7.2

$$Q = m_{ice}c_{ice}\Delta T_{ice} + m_{ice}L + m_{ice}c_{water}\Delta T_{water}$$
$$= 12(2.1 \times 4 + 340 + 4.2 \times 60)$$
$$= 7.2 \text{ kJ}$$

Question 15

在标准大气压下, 273 K 的空气被绝热压缩至原来体积的一半。计算空气温度的改变量。取空气摩尔比热的比值 $\gamma = 1.4$ 。答案以 $^\circ\text{C}$ 为单位。

Air at 273 K is compressed adiabatically to half of its volume at standard pressure. Calculate the change in its temperature. Take the ratio of the molar specific heat of air $\gamma = 1.4$. **Give the answer in unit $^\circ\text{C}$.**

Ans : 87

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$
$$T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1}$$
$$= 273 \left(\frac{1}{0.5} \right)^{1.4-1} = 360.22 \text{ K}$$
$$\Delta\theta = 360.22 - 273 = 87 \text{ K}$$

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Question 16

在双缝干涉实验中，屏幕上的第二亮纹与中心亮纹的距离为 3.0 cm。双缝的间距为 0.03 mm，与屏幕相距 2.0 m。求光的波长。答案以 μm 为单位。

In a double slit experiment, the second bright fringe occurs at 3.0 cm from the central bright spot. The slits are 0.03 mm apart and the screen is 2.0 m from the slits. Find the wavelength of the light. **Give the answer in unit μm .**

Ans: 0.23

$$d \sin \theta = n\lambda$$

Because $d \ll L$, $\sin \theta \approx \tan \theta = \frac{y}{L}$, so,

$$\lambda = \frac{dy}{nL} = \frac{(0.00003)(0.03)}{2(2.0)} = 2.3 \times 10^{-7} \text{ m} = 0.23 \times 10^{-6} \text{ m}$$

Question 17

警车的警报器发出频率为 500 Hz 的正弦波。已知空气中的声速为 340 ms^{-1} 。若警报器以 20 ms^{-1} 的速率相对空气移动，求在警报器后方的声波波长。答案以 m 为单位。

A police car's siren emits sinusoidal waves with frequency 500 Hz. The speed of sound in air is 340 ms^{-1} . Find the wavelengths of the sound waves behind the siren if it is moving at 20 ms^{-1} relative to the air. **Give the answer in unit m.**

Ans: 0.72

$$\lambda = \frac{v + v_s}{f} = \frac{340 + 20}{500} = 0.72 \text{ m}$$

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Question 18

一质量为 $7.7 \times 10^{-3} \text{ g}$ ，电荷为 $5.0 \times 10^{-6} \text{ C}$ 的小球以一根绝缘细线挂在具有均匀水平电场的区域。与小球相比，细线的质量可以忽略不计。平衡时，细线与垂直面成 60° 角。求电场的大小。答案以 Vm^{-1} 为单位。

A small sphere with mass $7.7 \times 10^{-3} \text{ g}$ and charge $5.0 \times 10^{-6} \text{ C}$ hangs from an insulated thread in a region with uniform and horizontal electric field. The mass of the thread is negligible compared with the sphere. When equilibrium, the thread forms an angle 60° to the vertical. Find the magnitude of the electric field. Give the answer in unit Vm^{-1} .

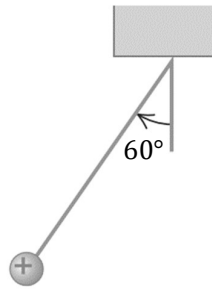


Figure 6

Ans: 26

Force balance gives

$$T \sin \theta - qE = 0$$

$$T \cos \theta - mg = 0$$

Combine the two equations

$$\tan \theta = \frac{qE}{mg}$$

$$E = \frac{mg}{q} \tan \theta = \frac{7.7 \times 10^{-6} \times 9.8}{5.0 \times 10^{-6}} \tan 60^\circ = 26 \text{ Vm}^{-1}$$

Question 19

一半径为 1.0 cm ，带有电荷 $-28 \mu\text{C}$ 的金属球被固定在一真空室中。一质量为 $3.0 \times 10^{-6} \text{ g}$ 带有电荷 $7.8 \mu\text{C}$ 的小粒子自离金属球中心 60 cm 处以速率 $0.40 \times 10^6 \text{ ms}^{-1}$ 射向金属球中心。求粒子在抵达金属球表面前的速率。答案以 10^6 ms^{-1} 为单位。

A metallic sphere with a radius 1.0 cm , carrying a net charge $-28 \mu\text{C}$ is held in stationary inside a vacuum chamber. A tiny particle with a net charge $q = 7.8 \mu\text{C}$ and mass $3.0 \times 10^{-6} \text{ g}$ is emitted towards the center of the metallic sphere with a speed of $0.40 \times 10^6 \text{ ms}^{-1}$ at a distance

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of 60 cm from the center of the sphere. Find the speed of the particle just before it reaches the surface of the sphere. **Give the answer in unit 10^6 ms^{-1} .**

Ans: 0.54

Using conservation of energy

$$\begin{aligned} \frac{1}{2}mv_f^2 + \frac{1}{4\pi\epsilon_0} \frac{Qq}{r_f} &= \frac{1}{2}mv_i^2 + \frac{1}{4\pi\epsilon_0} \frac{Qq}{r_i} \\ v_f^2 &= v_i^2 + \frac{2}{m} \frac{Qq}{4\pi\epsilon_0} \left(\frac{1}{r_i} - \frac{1}{r_f} \right) \\ &= (0.4 \times 10^6)^2 + \frac{2}{3 \times 10^{-9}} \frac{28 \times 7.8 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12}} \left(\frac{1}{0.6} - \frac{1}{0.01} \right) \\ v_f^2 &= 2.887 \times 10^{11} \\ v_f &= 5.37 \times 10^5 \text{ m/s} \end{aligned}$$

Question 20

一电荷为 $3e$, 质量为质子 12 倍的粒子束垂直进入 0.23 T 的均匀水平磁场, 并弯曲成直径 95 cm 的半圆。计算粒子束的速率。答案以 10^6 ms^{-1} 为单位。

A vertical beam of particles that have charges of magnitude $3e$ and mass 12 times the proton mass enters a uniform horizontal magnetic field of 0.23 T and is bent in a semicircle of diameter 95 cm. Find the speed of the particle. **Give the answer in unit 10^6 ms^{-1} .**

Take mass of proton(质子质量) $1.7 \times 10^{-27} \text{ kg}$, and $e = 1.6 \times 10^{-19} \text{ C}$

Ans: 2.6

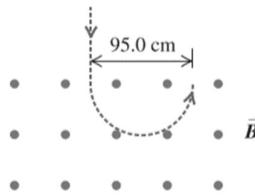


Figure 7

$$\begin{aligned} m \frac{v^2}{r} &= qvB \\ v &= \frac{q}{m} rB \\ &= \frac{3 \times 1.6 \times 10^{-19}}{12 \times 1.7 \times 10^{-27}} \frac{0.95}{2} \times 0.23 = 2.6 \times 10^6 \text{ m/s} \end{aligned}$$

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Question 21

一根 0.65 m 长的金属棒以 5.0 ms^{-1} 的稳定速度垂直于 0.73 T 的均匀磁场向右移动。该金属棒两端位于平行金属轨道上，轨道以一电阻为 25Ω 的电阻器连接。整个装置形成一个完整电路，如图 8 所示。金属棒和轨道的电阻可以忽略，计算电路的电流。答案以 mA 为单位。

A 0.65-m -long metal bar is pulled to the right at a steady speed 5.0 ms^{-1} perpendicular to a uniform 0.73 T magnetic field. The bar rides on parallel metal rails connected through a 25Ω resistor, so the apparatus forms a complete circuit as figure 8 shown. Ignore the resistance of the bar and the rails, calculate the current induced in the circuit. Give the answer in unit mA .

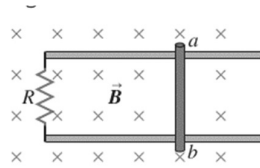


Figure 8

Ans: 95

$$I = \frac{emf}{R} = \frac{LvB}{R} = \frac{0.65(5.0)(0.73)}{25} = 95 \text{ mA}$$

Question 22

考虑一个电动势振幅为 15 V 的 RLC 串联电路，电阻 20Ω ，电感 1.2 mH 及电容 $4.0 \mu\text{F}$ 。在共振条件下，求电感两端电势差的振幅。答案以 V 为单位。

Consider an RLC serial circuit with emf amplitude 15 V , resistance 20Ω , inductance 1.2 mH , and capacitance $4.0 \mu\text{F}$. Find the amplitude of the voltage across the inductor at resonance. Give the answer in unit V .

Ans: 13

At resonance, $Z = R$,
The current

$$I = \frac{emf}{Z} = \frac{15}{20} = 0.75 \text{ A}$$
$$V_L = IX_L = IL \sqrt{\frac{1}{LC}}$$
$$= 0.75 \sqrt{\frac{1.2\text{m}}{4.0\mu}} = 13 \text{ V}$$

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Question 23

一内电阻为 $1.0\ \Omega$ 的安培计如图 9 接到一电路上以量测电流。求安培计的读数。答案以 mA 为单位。

An ammeter with an internal resistance of $1.0\ \Omega$ is inserted in the network to measure current as shown in the figure 9. Find the reading of the ammeter. **Give the answer in unit mA.**

Ans: 98

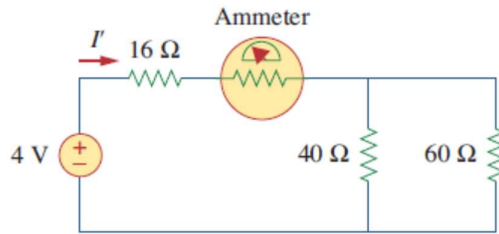


Figure 9

$$I\left(r + R_1 + \frac{R_2 R_3}{R_2 + R_3}\right) = V$$

$$I = \frac{4}{1 + 16 + \frac{40 \times 60}{40 + 60}} = 98\text{ mA}$$

Question 24

计算图 10 中外电路的等效电阻值。答案以 Ω 为单位。

Calculate the equivalent resistance in the circuit in figure 10. **Give the answer in unit Ω .**

Ans: 10

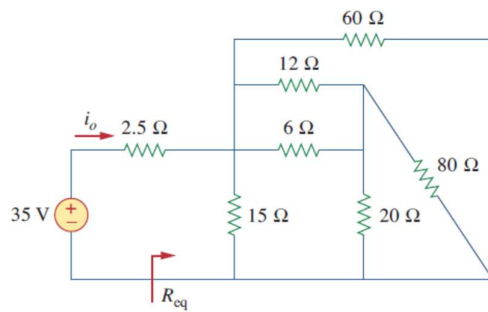


Figure 10

$$20 \parallel 80 = \frac{20 \times 80}{20 + 80} = 16$$

$$6 \parallel 12 = \frac{6 \times 12}{6 + 12} = 4$$

$$(4 + 16) \parallel 60 = \frac{20 \times 60}{20 + 60} = 15$$

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$$15 \parallel 15 = \frac{15 \times 15}{15 + 15} = 7.5$$
$$R_{eq} = 2.5 + 7.5 = 10 \Omega$$

Question 25

两电容器 $C_1 = 25 \mu F$ 及 $C_2 = 75 \mu F$ 如图 11 所示接到一个 100 V 的电源。求两电容充满电后的总能量。答案以 mJ 为单位。

Two capacitors $C_1 = 25 \mu F$ and $C_2 = 75 \mu F$ are connected to a 100-V source as shown in figure 11. Find the total energy stored in the capacitors when they are fully charged. **Give the answer in unit mJ.**

Ans: 94

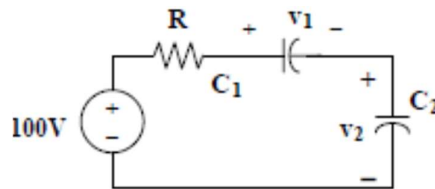


Figure 11

$$Q = C_1 v_1 = \frac{C_1 C_2}{C_1 + C_2} V$$
$$v_1 = \frac{75}{25 + 75} 100 = 75V$$
$$v_2 = 100 - 75 = 25V$$
$$E = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$
$$= \frac{1}{2} (25\mu 75^2 + 75\mu 25^2) = 94 mJ$$

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Question 26

考虑两个半径为 $4.0 \mu\text{m}$, 由锇元素组成的小球(锇是自然界中密度最大的物质, 其密度为 22.59 g/cm^3), 两球心水平相隔 $10 \mu\text{m}$, 被置放到离地心 6370 km 处。若 F_{mut} 表示两小球相互的引力大小, F_E 表示其中一小球受地球引力在两球心连线上的分量。计算比值 F_{mut}/F_E 。

Consider two Osmium spheres (the densest naturally occurring material, density 22.59 g/cm^3), each of $4.0 \mu\text{m}$ radius, separated horizontally by $10 \mu\text{m}$. Both spheres are at 6370 km from the center of the Earth. Let F_{mut} be the mutual gravitational force between the two spheres; F_E be the component of Earth's gravitational force on one of the spheres along the line joining the spheres. Calculate the ratio F_{mut}/F_E .

Mass of Earth 地球质量 $6.0 \times 10^{24} \text{ Kg}$, Universal gravitational constant 万有引力常数 $6.7 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

Ans: 0.52, 0.53

The mutual gravity is given by $F_{mut} = G \frac{m^2}{d^2}$, where mass is $m = \rho \frac{4}{3} \pi r^3 = 6.056 \times 10^{-12} \text{ kg}$ and the distance $d = 10^{-5} \text{ m}$. Hence $F_{mut} = 2.46 \times 10^{-23} \text{ N}$.

$$F_E = G \frac{mM}{R^2} \frac{d}{2R} = 4.71 \times 10^{-2} \text{ N},$$

$$\frac{F_{mut}}{F_E} = 0.52$$

The mutual gravity you will find is comparable to the one experienced by stars exhibiting **dark matter** effects. Yet, such an experiment would be demanding because your second calculation shows that non-uniformity of Earth's gravity has a contribution similar in magnitude.

Question 27

在理想条件下, 如果每秒吸收的光子少至 100 个, 正常人眼睛仍可侦测到波长 550 nm 的光。这相当于多少的功率? 答案以 10^{-17} W 为单位。

Under ideal conditions the normal human eye will record a visual sensation for wavelength 550 nm if as few as 100 photons are absorbed per second. What power level does this correspond to? Give the answer in unit 10^{-17} W .

Ans: 3.6

Energy of a photon with wavelength λ is given by $E = \frac{hc}{\lambda} = 3.61 \times 10^{-19} \text{ J}$.

To record the visual sensation the eye needs to absorb 100 photons per second.

The power absorbed is $P = 100 E = 3.61 \times 10^{-17} \text{ W}$.

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Question 28

钾金属片受距离 1.0 m 处 0.50 W 的灯泡照射。从钾表面移除一电子所需的能量为 2.1 eV。假设电子可吸收能量的面积近似于半径为 10^{-10} m 的圆形，光是从灯泡向各个方向发射的电磁波。问钾电子需要多长的时间才能吸收足够的能量从钾金属片中跳出来？

答案以 s 为单位。

Potassium plate is illuminated from 1.0 m by a 0.50 W bulb. The energy required to remove the electron from potassium surface is $W = 2.1$ eV. Assuming that the area of an electron to absorb energy is approximately the circle with radius 10^{-10} m and light is the electromagnetic wave emitted from the bulb in all directions, how long does it take for a potassium electron to absorb the energy sufficient to be ejected from the plate? **Give the answer in unit s.**

Ans :270

The bulb emits 0.5J in 1s into 4π angle. In 1 second the electron absorbs the energy proportional to the ratio of its surface to the area of the sphere with the radius of 1 meter, i.e. absorbed energy per second is

$$\begin{aligned} P_{\text{absorb}} &= \frac{\pi r^2}{4\pi R^2} P_{\text{emit}} \\ &= \frac{(10^{-10})^2}{4(1)^2} (0.5) \\ &= 0.125 \times 10^{-20} \text{ W} \end{aligned}$$

The energy needed to be ejected is $W = 2.1 \text{ eV} = 3.36 \times 10^{-19} \text{ J}$.

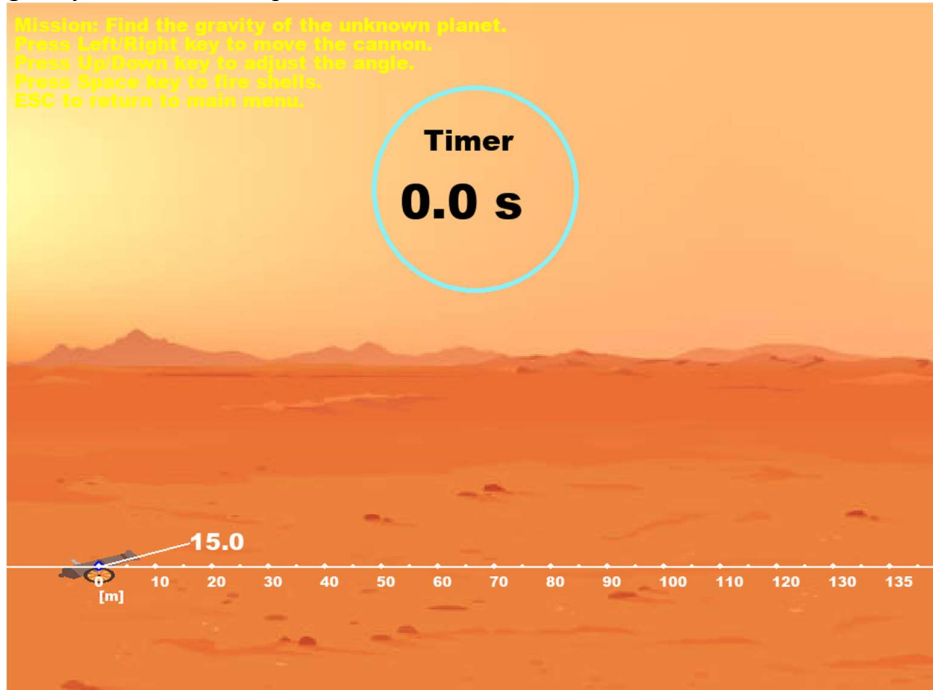
Hence the time required is $3.36 : 0.125 = 270$ s. Interestingly, in the experiment it takes practically no time to start the photocurrent.

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Question 29

[visuallabv3 \(xmumphys.com\)](http://visuallabv3.xmumphys.com)

Find the gravity of the unknown planet. Give the answer in unit ms^{-2} .



Use $v = \frac{x}{\cos \theta t}$ to find initial speed. Then use $R = \frac{v^2 \sin 2\theta}{g}$ to find gravity.

Example. When $\theta = 45^\circ$, $x \approx 69 \text{ m}$, $t = 5.1 \text{ s}$

$$v = 19.1 \text{ m/s}$$

$$g = \frac{v^2 \sin 2\theta}{x} = 5.3 \text{ m/s}^2$$

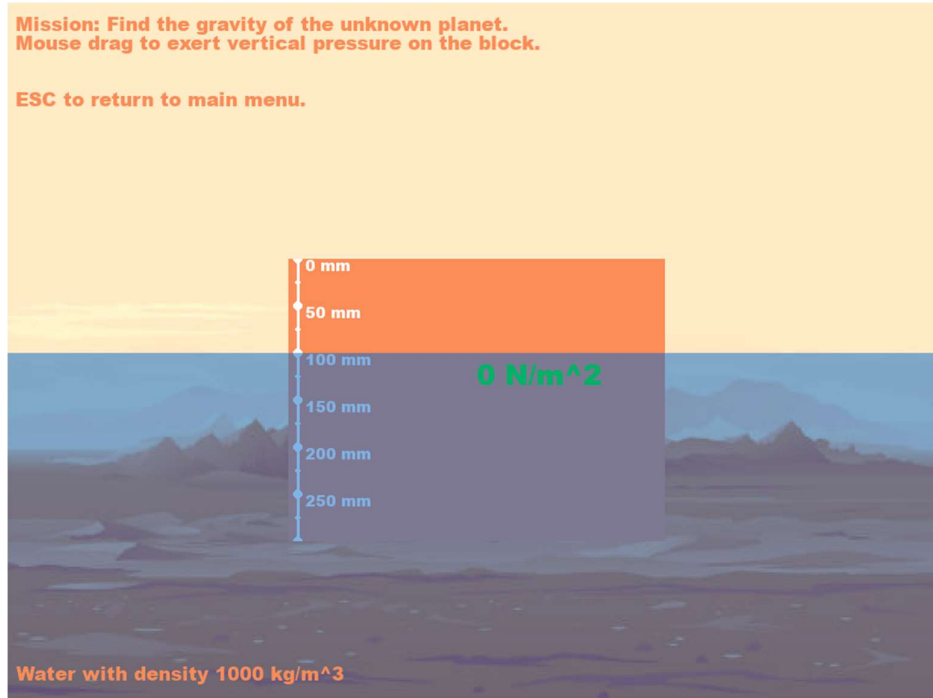
We accept answer 5.2, 5.3, and 5.4

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Question 30

[visuallabv3 \(xmumphys.com\)](http://visuallabv3(xmumphys.com))

Find the gravity of the unknown planet. Give the answer in unit ms^{-2} .



Use $F = \rho_w V_w g - \rho_b V_b g$,

When equilibrium, $\rho_b V_b g = \rho_w V_w g \Rightarrow \rho_b = \frac{2}{3} \rho_w$

$$F/A = \rho_w h_w g - \rho_b h_b g$$

$$g = \frac{\frac{F}{A}}{\rho_w h_w - \frac{2}{3} \rho_w h_b}$$

Example:

When $h_w = 0.15\text{m}$, $\frac{F}{A} = -600 \text{ N/m}^2$

$$g = \frac{-600}{1000(0.15 - \frac{2}{3}0.3)} = 12 \text{ m/s}^2$$

***** End of Paper *****

****试卷结束****

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