2021 厦大杯中学物理比赛
2021 XMUM Cup Physics Competition for Secondary School

Question	Acceptable answers		Question	Acceptable answers
1	1.4		15	1.3
2	1.2		16	4.5
3	7.6		17	0.25
4	2.3		18	1.4
5	2.5		19	1.6
6	3.2		20	16
7	55		21	5.6, 13
8	15		22	4.2
9	420		23	0.15
10	0.25		24	0.45
11	67		25	0.45,0.49
12	56		26	0.21,0.20,0.2
13	5.5,5.0		27	1.6
14	8.3		28	2.4

Topics	Questions	No. Questions
Mechanics	1,2,3,4,5,6,7,8,9,26	10
Thermodynamics	10,11,12,13,14,15	6
Electromagnetic	16,17,18,19,20,21,22	7
Modern Physics	24,25,28	3
Optics	25,23	2

Question 1.

A car moving with constant acceleration covers a distance of 70.0 m in 7.00 s. The speed of the car after it travelled 70.0 m is 15.0 m s^{-1}. What is the magnitude of the acceleration of the car? **Give the answer in unit of m s^{-2} to 2 significant figures.**

一辆汽车在 7.00 s 内匀加速行驶了 70.0 m 的距离。若汽车在到达 70.0 m 后的速率为 15.0 m s^{-1}, 则其加速度大小是多少？**答案以 m s^{-2} 为单位取 2 位有效数字。**

Solution: *Given $s = 70.0$ m, $t = 7.00$ s, and u is unknown.*

$$s = ut + \frac{1}{2}at^2$$
$$70 = 7u + \frac{a}{2}7^2$$
$$20 = 2u + 7a \qquad (1)$$

Taking the other kinematic equation,

$$v = u + at$$
$$15 = u + 7a$$
$$30 = 2u + 14a. \qquad (2)$$

Taking the difference between Eq. (1) and (2),

$$10 = 7a$$
$$a = \frac{10}{7} = 1.43\ \mathrm{m\,s^{-2}}.$$

Question 2

A mass 0.60 kg is attached to a spring of spring constant 4.00 N m^{-1}. The natural length of the spring is 0.90 m. One end of the spring is fixed on a horizontal table and the mass is set to move in a circular motion of radius, r with speed $\sqrt{2.50}$ m s^{-1}. What is the radius, r of the circle? **Give the answer in unit of m to 2 significant figures.**

一弹簧的劲度系数为 4.00 N m^{-1}, 自然长度为 0.90 m。弹簧的一端固定在水平桌上，另一端系有质量 0.60 kg。当质量以$\sqrt{2.50}$ m s^{-1} 的速率作圆周运动时，圆轨迹的半径 r 应为多少？**答案以 m 为单位取 2 位有效数字。**

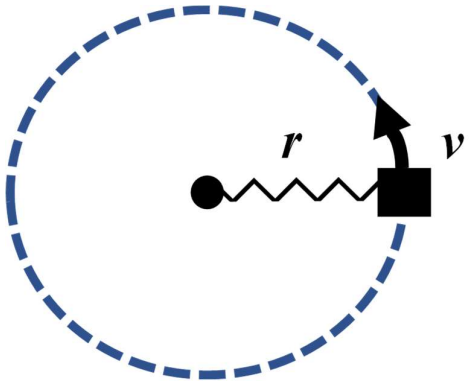


Figure 1.

Solution:

The mass is in circular motion where the centripetal force provided by the spring

$$F = ma$$

$$kx = m\frac{v^2}{L + x}$$

$$kx^2 + kLx - m\,v^2 = 0$$

$$(4)x^2 + (4)(0.9)x - (0.6)(2.5) = 0$$

$$x = \frac{-3.6 \pm \sqrt{3.6^2 + 4(4)(1.5)}}{8} = 0.31$$

Taking the positive root, the radius of the circle is $x + L = 0.31 + 0.90 = 1.2\ m$

Question 3

Given that the radius of the earth is 6.38 x10^6 m, the mass of the earth is 5.98 x 10^{24} kg and the universal gravitational constant G = 6.67 x 10^{-11} N$\cdot m^2kg^{-2}$. Calculate the speed of the Hubble space telescope orbiting at a height of 520 km above the earth's surface. Assume the orbit is uninform circular motion. **Give the answer in unit of km s^{-1} to 2 significant figures.**

已知地球半径为 6.38 x10^6 m，地球质量为 5.98 x 10^{24} kg， 万有引力常数G = $6.67 \times 10^{-11}\text{N} \cdot \text{m}^2\text{kg}^{-2}$。哈勃天文望远镜在离地面 520 km 高的圆轨道匀速运动，计算哈勃天文望远镜的速率。**答案以 km s^{-1} 为单位取 2 位有效数字。**

Solution:

The centripetal force is provided by gravitational force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.38 + 0.52) \times 10^6}}$$

$$v = 7.6\ km\ s^{-1}$$

Question 4

A sailor strikes the side of the ship just below the water line. He hears the echo of the sound reflected from the ocean floor 3.0 s later. Calculate the depth of the sea at that position. Assume the speed of sound in sea water is 1534 m s^{-1}. **Give the answer in unit of km to 2 significant figures.**

某水手敲打船身与水面齐高的位置。他在 3.0s 后听到来自海底的反射声波。若声音在海水中的传播速度为 1534 m s^{-1}， 估算在船位置海水的深度。**答案以 km 为单位取 2 位有效数字。**

Solution:

$$s = vt = 1534 \times \frac{3.0}{2} = 2301\ m$$

$$s = 2.3\ \ km$$

Question 5

Two particles move in a uniform gravitational field. The field strength is 9.8 N kg^{-1} vertically downward. At the initial moment, the particles were located at the same position and moved with velocity 3.5 m s^{-1} horizontally in opposite directions. Find the horizontal distance between the particles at the moment when their velocity vectors become mutually perpendicular. **Give the answer in unit of m to 2 significant figures.**

两个粒子在匀强引力场中运动。引力场强度为 9.8 N kg^{-1} 垂直向下。 在初始时刻，两粒子处于同一个位置并以 3.5 m s^{-1} 的速度沿相反方向水平移动。 求当两粒子的速度向量相互垂直时，粒子之间的水平距离。**答案以 m 为单位取 2 位有效数字。**

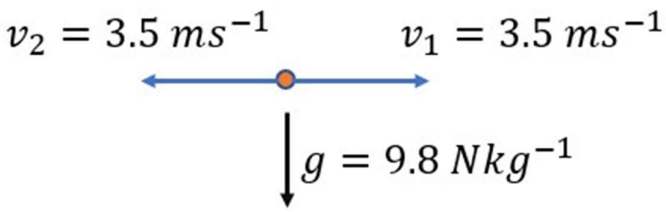


Figure 5

Answer: 2.5 meters

Solution:

Velocity of particle 1 $\vec{v}_1 = 3.5\,\hat{\imath} - gt\,\hat{\jmath}$

Velocity of particle 2 $\vec{v}_2 = -3.5\,\hat{\imath} - gt\,\hat{\jmath}$

When $\vec{v}_1 \perp \vec{v}_2$, $\vec{v}_1 \cdot \vec{v}_2 = \mathbf{0}$

$$-3.5^2 + g^2t^2 = 0$$

$$t = \frac{3.5}{9.8}s$$

The distance between the two particles is, $[3.5 - (-3.5)] \times \frac{3.5}{9.8} = 2.5\ m$

Question 6

A 0.150-kg glider P is moving to the right on a frictionless, horizontal air track with a speed of 0.80 m s^{-1}. It has a head-on elastic collision with glider Q that is moving to the left with a speed of 2.20 m s^{-1}. The mass of glider Q is 0.30 kg. Find the final speed of the glider P after collision. **Give the answer in unit of** m s^{-1} **to 2 significant figures.**

一 0.15-kg 的 P 滑块以 0.80 m s^{-1} 的速率在无摩擦的滑轨上往右移动， 与另一以速率 2.20 m s^{-1} 往左移动的物体 Q 发生弹性正撞。 物体 Q 的质量为 0.30 kg, 求滑块 P 在碰撞后的速率。**答案以** m s^{-1} **为单位取 2 位有效数字。**

Solution: *Given* $m_1 = 0.150$ kg, $m_2 = 0.300$ kg, $u_1 = 0.80\ \mathrm{m\,s^{-1}}$, *and* $u_2 = -2.20\ \mathrm{m\,s^{-1}}$. *The elastic collision formula is*

$$\begin{aligned} u_1 + v_1 &= u_2 + v_2 \\ 0.8 + v_1 &= -2.2 + v_2 \\ v_2 &= v_1 + 3 \end{aligned} \tag{3}$$

Conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{4}$$

$$0.15(0.8) + 0.3(-2.2) = 0.15v_1 + 0.3v_2 \tag{5}$$

Substitute Eq. (3) into (5),

$$\begin{aligned} -0.54 &= 0.15v_1 + 0.3(v_1 + 3) \\ -0.54 &= 0.45v_1 + 0.9 \\ v_1 &= \mathbf{-3.2\ m\,s^{-1}}. \end{aligned}$$

Question 7

Two planets of masses 3M and 2M are at distance d =100 km away from each other. An asteroid of mass m is on the line between the two planets, and it is in force equilibrium, as shown in Fig. 2. Find the distance, r of the asteroid from the planet 3M. **Give the answer in unit of km to 2 significant figures.**

两行星相距 d = 100 km，质量分别为 3M 和 2M。一质量为 m 的陨石处在两行星的连线上刚好处于力平衡状态，如图 2 所示。求陨石离 3M 行星的距离，r。**答案以 km 为单位取 2 位有效数字。**

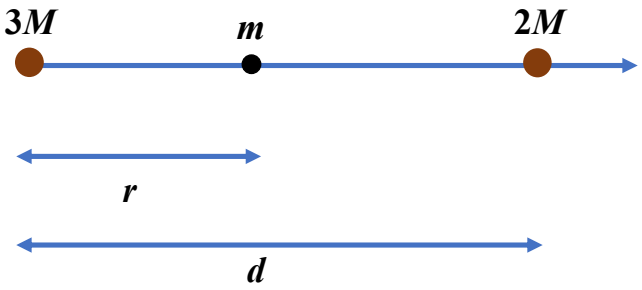


Figure 7.

Solution: *Taking the unit vector in the horizontal to be the $\hat{\imath}$, the net force due to $3M$ and $2M$ are*

$$\begin{aligned}\vec{F} &= -\frac{3GMm}{r^2}\hat{\imath} + \frac{2GMm}{(d-r)^2}\hat{\imath} \\ &= GMm\left(\frac{2}{(d-r)^2} - \frac{3}{r^2}\right)\hat{\imath}.\end{aligned}$$

For m to be at equilibrium,

$$\begin{aligned}\frac{3}{r^2} &= \frac{2}{(d-r)^2} \\ 3(d-r)^2 &= 2r^2 \\ r^2 - 6dr + 3d^2 &= 0.\end{aligned}$$

The solution to the quadratic equation is $r = \left(3 \pm \sqrt{6}\right) d$. The physically relevant solution corresponds to the lower sign. For $d = 100$ km, the position of r in equilibrium is

$$\begin{aligned}r &= \left(3 - \sqrt{6}\right) d \\ &= 55 \text{ km}.\end{aligned}$$

Question 8

A car starts moving from rest with acceleration a = 5.0 m s^{-2}, then with constant velocity, and finally, decelerating at the same rate, 5.0 m s^{-2}, comes to stop. The total time of motion is 25s. The average velocity during the 25s is 72 km per hour. How long does the car move with constant velocity? **Give the answer in unit of s to 2 significant figures.**

一辆汽车从静止开始以加速度 5.0 m s^{-2} 行驶，然后以恒定速度移动一段时间后再以 5.0 m s^{-2} 的减速度减速至停止。 已知运动的总时间为 25 s，且 25s 内的平均速度为 72 公里每小时。 求汽车以恒定速度移动的时间。**答案以 s 为单位取 2 位有效数字。**

Answer: 15 seconds

Solution:

Let the time moving in constant velocity is Δt

The total displacement is

$$s = \frac{1}{2}a\left(\frac{t-\Delta t}{2}\right)^2 \times 2 + v\,\Delta t \;\; = \frac{a}{4}(t-\Delta t)^2 + a\,\frac{t-\Delta t}{2}\Delta t$$

The average velocity is

$$\frac{s}{t} = \frac{a(t-\Delta t)^2 + 2at\Delta t - 2a\,\Delta t^2}{4t}$$

$$\Delta t = t\sqrt{1-\frac{4v}{at}} = 25\sqrt{1-\frac{(4)(20)}{(5)(25)}}$$

$$\Delta t = 15\ s$$

Question 9

A vertical cylinder closed from both ends is equipped with an easily moving piston dividing the volume into two parts, each containing one mole of air. In equilibrium at $T_0 = 298.7$ K, the volume of the upper part is 4.0 times of the lower part. At what temperature will the ratio of these volumes be equal to 3.0? **Give the answer in unit of K to 2 significant figures.**

一个两端封闭的垂直圆柱容器中配有一个易于移动的活塞。活塞将容器体积分成上下两部分，每部分各含有一摩尔的空气。 在平衡状态下气体温度为 298.7 K， 上部的体积是下部分体积的 4.0 倍。 求在什么温度下，上下体积的比值等于 3.0？**答案以 K 为单位取 2 位有效数字。**

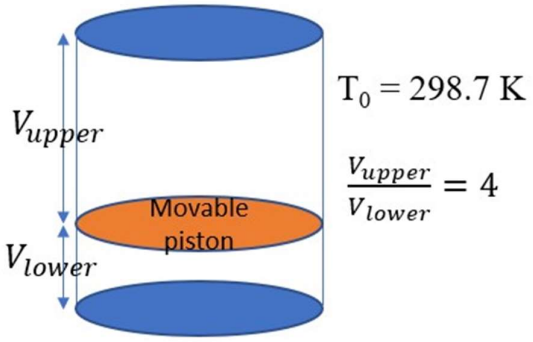


Figure 9.

Answer: 420 K

Let the upper part pressure be P_1, and the lower part pressure be P_2

the mass of piston is m, area is A, and initial temperature is T_0 and volume is $5V_0$

when equilibrium

$$\boldsymbol{P_1}A + mg = P_2A$$

$$\frac{mg}{A} = P_2 - P_1$$

Using ideal gas law

$$\frac{mg}{A} = \frac{RT_0}{V_0} - \frac{RT_0}{4V_0} = \frac{RT}{V} - \frac{RT}{3V}$$

$$T = \frac{\left(1 - \frac{1}{4}\right)}{1 - \frac{1}{3}} T_0 \frac{V}{V_0}$$

The total volume is constant

$$V + 3V = 5V_0$$

$$\frac{V}{V_0} = \frac{5}{4}$$

$$T = \frac{3}{4} \times \frac{3}{2} \times 298.7 \times \frac{5}{4} = 420K$$

Question 10

Gaseous hydrogen at pressure of 100 kPa and temperature 300 K was sealed in a vessel of volume 0.005 m^3. After the temperature dropped $60K$, what amount of heat would loss from the gas. Consider the gas as an ideal gas, the ratio of specific heat capacity at constant pressure and volume is 1.4. **Give the answer in unit of kJ to 2 significant figures.**

气态氢在气压 100 kPa 及温度 300 K 下封存在体积 0.005 m^3 的容器中。当温度下降了 $60K$，气体会损失多少热量? 气态氢可视为理想气体，等压与等容热容比为 1.4。**答案以 kJ 为单位取 2 位有效数字。**

Answer: 250 J

Solution:

By using the first law of thermodynamics

$$Q = \Delta U + W$$

$$Q = nC_V \Delta T + 0$$

$$Q = \frac{nR\Delta T}{\gamma - 1}$$

Ideal gas law

$$n = \frac{PV}{RT}$$

$$Q = \frac{PV}{T}\frac{\Delta T}{\gamma - 1}$$

$$Q = \frac{100000 \times 0.005}{300}\frac{60}{1.4 - 1} = 0.25\ kJ$$

Question 11

A hot copper pot of mass 2.0 kg is at temperature 90 ˚C. Cool water of 0.10 kg at 25 ˚C is poured into the pot. Assuming no heat is lost to the surroundings, calculate the final temperature of the water after it has reached equilibrium. The specific heat of copper is 390 J kg^{-1} K^{-1} and the specific heat of water is 4190 J kg^{-1} K^{-1}. **Give the answer in unit of °C to 2 significant figures.**

一铜制热壶的质量为 2.0 kg，温度为 90 ˚C。倒入 0.10 kg， 25 ˚C 的水后，假设没有热损失，计算热平衡后，水的温度。已知铜的比热为 390 J kg^{-1} K^{-1}，水的比热为 4190 J kg^{-1} K^{-1}。**答案以°C为单位取 2 位有效数字。**

Solution: *Take the copper pot to be system 1 and the water as system 2.*

$$-Q_1 = Q_2$$
$$-m_1 c_1 (T_f - T_1) = m_2 c_2 (T_f - T_2)$$
$$-(2)(390)(T_f - 90) = (0.10)(4190)(T_f - 25)$$
$$T_f = \mathbf{67}^\circ\mathrm{C}.$$

Question 12

A 200 g sample is placed in a cooling apparatus that removes energy as heat at constant rate. Figure 12. gives the temperature T of the sample versus time t. The sample freezes during the process. If the specific heat of the sample in liquid phase is 2500 J kg^{-1} K^{-1} , find the fusion heat of the sample. **Give the answer in unit of J/g to 2 significant figures.**

200g 的样品被置放在一冷却装置，样品的能量会以热的形式等速率被移除。图 12 显示样品的温度对时间的关系图。样品在冷却过程中凝固。已知样品在液态时的比热为 2500 J kg^{-1} K^{-1}。试求样品的熔解热。**答案以 J/g 为单位取 2 位有效数字。**

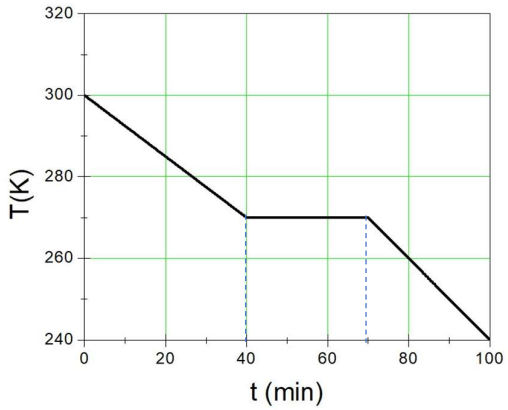


Figure 12

Solution:

$$\frac{dQ}{dt} = mc\frac{dT}{dt} = (0.2)(2500)\frac{(30)}{40} = 375\ J/min$$

$$L = \frac{1}{m}\frac{dQ}{dt}\Delta t = \frac{1}{200}(375)(30) = 56\ J/g$$

Question 13

An ideal gas expands isothermally, performing 5.5×10^3 J of work in the process. Calculate the heat absorbed during this expansion. **Give the answer in unit of kJ to 2 significant figures.**

一理想气体在等温膨胀过程中做功 5.0×10^3 J。计算过程中吸收的热。**答案以 kJ 为单位取 2 位有效数字。**

Solution:

By using the first law of thermodynamics

$$Q = \Delta U + W$$

Isothermal process

$$Q = 0 + W = 5.5\ kJ$$

***Note: accept both 5.5 and 5.0 as the answer.**

Question 14

Calculate the total heat needed to convert 2.74 kg of ice at -10 ˚C to steam at 100 ˚C. Given that specific heat of ice 2050 J kg^{-1} K^{-1}, specific heat of water 4184 J kg^{-1} K^{-1}, heat of fusion of water 334 kJ kg^{-1} and heat of vaporization of water 2257 kJ kg^{-1}. **Give the answer in unit of 10^6J to 2 significant figures.**

计算 2.74 kg，温度为-10 ˚C 的冰完全转化成 100 ˚C 的蒸气所需的热。已知冰和水的比热容量分别为 2050 J kg^{-1} K^{-1} 及 4184 J kg^{-1} K^{-1}。熔解热和汽化热分别为 334 kJ kg^{-1} 及 2257 kJ kg^{-1}。**答案以 10^6J 为单位取 2 位有效数字。**

Solution:

By using the first law of thermodynamics

$$Q_{ii} = 1(2050)(10) = 20.5\ kJ$$

$$Q_{iw} = 1(334k) = 334\ kJ$$

$$Q_{ww} = 1(4184)(100) = 418.4\ kJ$$

$$Q_{ws} = 1(2257k) = 2257\ kJ$$

$$Q = 2.74(20.5\ + 334 + 418.4 + 2257)kJ = 8.3 \times 10^6 J$$

Question 15

The global warming of climate change affects sea level through two ways. One is additional water due to melting glaciers that are on land. The other way is through thermal expansion of the ocean. The linear coefficient of thermal expansion of sea water is 1.7 x10^{-4} $^{o}C^{-1}$. The ocean has an average depth of 3850 m. If the entire water column increased by 2.0^{o}C, how much would sea level be increased? **Give the answer in unit of m to 2 significant figures.**

全球暖化会通过两种方式对海平面造成影响。第一种是陆地上的冰山融化增加了海水。另一种方式是海水因热而膨胀。已知海水的线性膨胀系数为 1.7 x10^{-4} $^{o}C^{-1}$。全球海洋的平均深度为 3850m。若海水温度上升 2.0^{o}C，海平面将上升多少？**答案以 m 为单位取 2 位有效数字。**

Solution:

$$\Delta h = \alpha h \Delta T = 1.7 \times 10^{-4} \times 3850 \times 2.0 = 1.3\ m$$

Question 16

A positive point charge of 50 μC is located in the plane x-y at the point with position vector $\vec{r}_0 = 2\hat{\imath} + 3\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are the unit vectors of the x and y axes in unit of meters. Find the electric field strength E at the point with position vector $\vec{r} = 8\hat{\imath} - 5\hat{\jmath}$.(take the electrostatic constant $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 N\ m^2 C^{-2}$) **Give answer in unit of kV/m to 2 significant figures.**

一 50 μC 的正电荷被置放在 x-y 平面上，位置向量为$\vec{r}_0 = 2\hat{\imath} + 3\hat{\jmath}$的点, 其中$\hat{\imath}$, $\hat{\jmath}$分别为 x, y 轴的单位向量，单位为 m。求在位置向量$\vec{r} = 8\hat{\imath} - 5\hat{\jmath}$处 的电场强度。(取静电常数 $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 N\ m^2 C^{-2}$) **答案以 kV/m 为单位取 2 位有效数字。**

Answer: 4.5 kV/m

Solution:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r}_0|^2}$$

$$E = 9.0 \times 10^9 \frac{50\ \mu}{|8\hat{\imath} - 5\hat{\jmath} - 2\hat{\imath} - 3\hat{\jmath},|^2} = 4.5\ kV/m$$

Question 17

A point charge Q is placed at the mid-point of an edge of the cube of length a as shown in Fig. 17. The total electric flux through the cube is $\alpha \frac{Q}{\varepsilon_0}$. What is the value of α?

一点电荷放置于立方体某一边上的中点，立方体的边长为 a, 如图所示。通过立方体的电场通量为 $\alpha \frac{Q}{\epsilon_0}$。$\alpha$的值为多少?

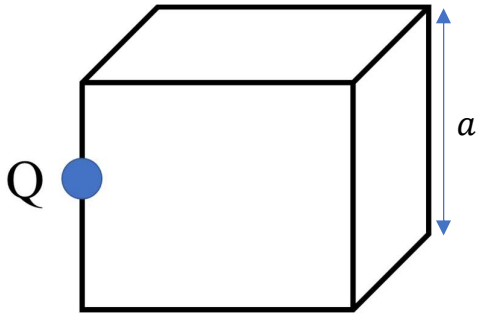


Figure 17.

The flux through the shaded part is

$$\Phi_E = \frac{1}{4} \times \textit{flux through the larger cube}$$
$$= \frac{1}{4}\frac{Q}{\epsilon_0}$$

Therefore, $\mathbf{r} = \frac{1}{4}$

Question 18

A hair dryer is connected to an ac source of voltage $V = 200\sin(120\pi t)$ in unit of Volt(V). Assume the hair dryer had a resistance of 100 Ω, calculate the rms current. **Give the answer in unit of A to 2 significant figures.**

一吹风筒连接到交流电压 V = 200 sin (120π*t*)。 电压的单位为伏特(V)。假设吹风筒的电阻为 100 Ω, 计算电流方均根值。**答案以 A 为单位取 2 位有效数字。**

Solution:

From $V = 200\sin(120\pi t)$

$$V_0 = 200\ V$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{200}{\sqrt{2}}\frac{1}{100} = 1.4\ A$$

Question 19

A stereo is connected to a remote speaker with an aluminium wire of length 30 m. The resistivity of the aluminium wire is 2.65 x 10^{-8} Ω m. Calculate the radius of the wire should be used in order to keep the resistance of the wire less than 0.1 Ω. **Give the answer in unit of *mm* to 2 significant figures.**

远处的喇叭以 30m 长的铝导线与音响系统连接。铝导线的电阻率为 2.65 x 10^{-8} Ω m，计算铝导线的半径应为多少才能使电阻值维持在 0.1 Ω。**答案以 mm 为单位取 2 位有效数字。**

Solution:

$$R = \frac{\rho L}{A}$$

$$0.10 = 2.65\ \text{x}\ 10^{-8} \times \frac{30}{\text{A}}$$

$$\pi r^2 = 2.65\ \text{x}\ 10^{-8} \times \frac{30}{0.10}$$

$$r = 1.6\ mm$$

Question 20

A conducting sphere of radius 4.00 cm carrying charge 32.0 C is brought into contact with another uncharged conducting sphere of radius 12.0 cm. Then the two spheres are separated. What is the difference of the charges between the two spheres? **Give the answer in unit of C to 2 significant figures.**

一半径 4.00cm 的导体球带有 32.0 C 的电量，与另一半径为 12.0 cm 不带电的导体球接触后被分开。两带电球的电量相差多少？**答案以 C 为单位取 2 位有效数字。**

Solution: *Let $Q_0 = 32$ cm be the initial total charge, which is conserved. When the two spheres touch, charges will distribute until both surfaces are at equipotential,*

$$V_1 = V_2$$
$$\frac{Q_1}{\cancel{4\pi\epsilon_0} R_1} = \frac{Q_2}{\cancel{4\pi\epsilon_0} R_2}$$
$$Q_1 = \frac{R_1}{R_2} Q_2.$$

By the conservation of charges,

$$Q_1 + Q_2 = Q_0$$
$$\left(1 + \frac{R_1}{R_2}\right) Q_2 = Q_0$$
$$\left(1 + \frac{4}{12}\right) Q_2 = 32.0 \text{ C}$$
$$Q_2 = \mathbf{24.0} \text{ C}.$$

The difference of the two charges is 24-8 = 16

Question 21

An insulating sphere with radius 0.10 m has 0.9 nC of charge uniformly distributed throughout its volume. The centre of the sphere is 0.24 m above a large uniform sheet that has charge density -8.0 nC m^{-2}. The point where the electric field is zero is x cm above the centre of the sphere. Find the value of x. **Give the answer to 2 significant figures.**

一绝缘球的半径为 0.10 m, 0.9 nC 的电量均匀分布在球体内。球体的中心离一无限大均匀带电的平板 0.24 m，平板带电密度为−8.0 nC m^{-2}。在球心上方 x cm 处的电场恰为零。 求 x 的值。**答案取 2 位有效数字。**

Solution: *The magnitude of electric field inside the sphere is*

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}.$$

The direction of the electric fields from the sphere are radial. So only the fields directly above the centre may cancel the fields from the sheet,

$$E_{\text{tot}} = \frac{Qr}{4\pi\epsilon_0 R^3} - \frac{\sigma}{2\epsilon_0}$$
$$0 = \frac{1}{2\epsilon_0}\left(\frac{Qr}{2\pi R^3} - \sigma\right)$$
$$r = 2\,\pi\frac{R^3\sigma}{Q} = \frac{2\pi(0.1)^3(8.0\times10^{-9})}{0.9\times10^{-9}} = 0.056m = 5.6\,cm$$

The magnitude of electric field outside the sphere is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$E_{tot} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} - \frac{\sigma}{2\epsilon_0} = 0$$

$$r = 0.13\ m = 13\ cm$$

Question 22

An electron in a particle accelerator has a speed of 6.8 x 10^6 ms^{-1}. The electron encounters a magnetic field of 0.50 T and whose direction makes an angle of θ = 45˚ with respect to the electron's velocity. Calculate the magnitude of acceleration of the electron. Given that the mass of the electron is 9.11 x 10^{-31} kg and the charge of the electron is -1.60 x 10^{-19} C. **Give the answer in unit of 10^{17}m s^{-2} to 2 significant figures.**

一电子在粒子加速器中的速率为 6.8 x 10^6 ms^{-1}。电子处在 0.50T 的匀强磁场中，速度方向与磁场方向夹 45˚角。求电子的加速度大小。已知电子质量为 9.11 x 10^{-31} kg，带电量为-1.60 x 10^{-19} C。**答案以 10^{17}m s^{-2} 为单位取 2 位有效数字。**

Solution:

$$F = qBv\sin\theta$$

$$a = \frac{F}{m} = \frac{qBv\sin\theta}{m}$$

$$a = \frac{1.60 \times 10^{-19} \times 0.50 \times 6.8 \times 10^6 \sin 45°}{9.11 \times 10^{-31}} = 4.2 \times 10^{17} m/s^2$$

Question 23

Angular separation of the interference fringes produced by a double-slit arrangement using sodium light (wavelength is 589 nm) is 0.20 °. What is the angular separation if the arrangement is immersed in water (refractive index is 1.33)? **Give the answer in unit of degree to 2 significant figures.**

一双缝实验装置使用钠光源(波长为 589 nm)产生亮纹的角间隔为0.20 °。若将此实验装置放到水中进行，则亮纹的角间隔变为多少？(水的折射率为 1.33) **答案以角度为单位取 2 位有效数字。**

Solution:

$$\sin\theta \approx \theta, \ \ when\ \theta \ll 1$$

In double-slit experiment

$$d\,\Delta\theta \approx \lambda$$

$$\Delta\theta = \frac{\lambda}{d} = \frac{\lambda_0}{dn} = \frac{0.20}{1.33} = 0.15$$

Question 24

What amount of energy should be added to an electron to reduce its de Broglie wavelength from 100 to 50 picometers? (The mass of an electron is 9.11 x 10^{-31} kg, Planck constant is 6.63 x 10^{-34} m^2 Kg s) **Give the answer in unit of keV to 2 significant figures.**

应该向电子添加多少能量以将其德布罗意波长从 100 pm 减少到 50 pm？(1.0 pm = 10^{-12} m, 电子质量为 9.11 x 10^{-31} kg，普朗克常数为 6.63 x 10^{-34} m^2 Kg s) **答案以 keV 为单位取 2 位有效数字。**

Answer: 0.45 keV

From $\lambda = \dfrac{2\pi\hbar}{p} = \dfrac{2\pi\hbar}{\sqrt{2mT}}$

we find $T = \dfrac{4\pi^2\hbar^2}{2m\lambda^2} = \dfrac{2\pi^2\hbar^2}{m\lambda^2}$

Thus $T_2 - T_1 = \dfrac{2\pi^2\hbar^2}{m}\left(\dfrac{1}{\lambda_2^2} - \dfrac{1}{\lambda_1^2}\right)$

Substitution gives $\Delta T = 451\ \text{eV} = 0{\cdot}451\ \text{keV}.$

Question 25

Passing down to the ground state, excited Ag^{109} nuclei emits either gamma quanta with energy 87 keV or K conversion electrons whose binding energy is 26 keV. The production of K conversion electrons can be considered as an internal photoelectric effect induced by the gamma quanta. Find the speed of K conversion electrons. (The rest energy of electron is 0.511 MeV) **Give the answer in unit of speed of light c to 2 significant figures.**

当激发态的 Ag^{109} 原子核跃迁回到基态时，会发射能量为 87 keV 的伽马量子或结合能为 26 keV 的 K 转换电子。 K 转换电子的产生可视为由伽马量子引发原子内部的光电效

应。试找出这些 K 转换电子的速率。 (电子的静止质量为 0.511 MeV) **答案以光速 c 为单位取 2 位有效数字。**

Answer: 0.45 c

In internal conversion, the total energy is used to knock out K electrons. The K.E. of these electrons is energy available-B.E. of K electrons

$$= (87 - 26) = 61 \text{ keV}$$

The total energy including rest mass of electrons is $0{\cdot}511 + 0{\cdot}061 = 0{\cdot}572$ MeV

The momentum corresponding to this total energy is

$$\sqrt{(0{\cdot}572)^2 - (0{\cdot}511)^2}\ / c = 0257 \text{ MeV/c.}$$

The velocity is then $\dfrac{c^2 p}{E} = c \times \dfrac{0{\cdot}257}{0{\cdot}572} = 0{\cdot}449\ c$

Accept also non relativistic approximation

$$v = \sqrt{\frac{2E}{m}} = \sqrt{2 \times \frac{61}{511}} = 0.49\ c$$

问题 26 至 28 为虚拟实验题, 参赛者须通过以下链接进入虚拟实验室。在虚拟实验室中, 按照题目的要求获取相关数据以求得正确答案。最后再回到此页, 将答案填入对应的空格中.

Questions 25 to 28 are virtual experimental questions. Participants must enter the virtual laboratory through the following link. In the virtual laboratory, obtain the relevant data according to the requirements of the question and use the data to calculate the correct answer. Finally, return to this page and fill the answers in the corresponding box.

Question 26

Objective: Determine the coefficient of maximum static friction between the metal and the board. **Keep 2 significant figures for your answer**.
In this virtual lab, you can
1. Hold 'c' to turn the crank clockwise.
2. Hold 'z' to turn the crank counter-clockwise.
3. Click 'reset' to reset the setup.

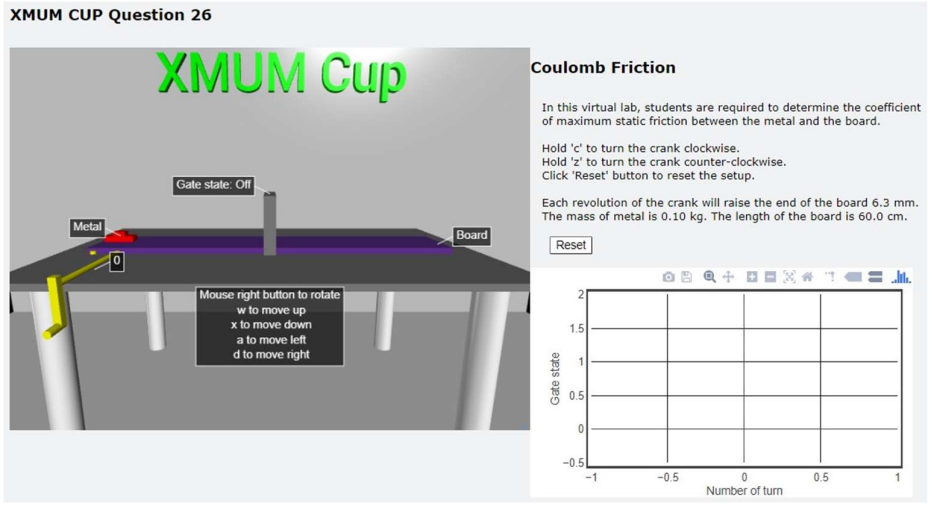
实验目的: 求金属块与平板间的最大静摩擦系数。答案取 2 位有效数字。
在此虚拟实验室中， 你可以
1.按键盘 c 键以顺时针转动曲柄,
2.按键盘 z 键以逆时针转动曲柄。
3.点击按钮 'Reset'恢复初始设定。

Maximum static friction is given by $f_{max} = \mu_s N$, where N is the normal force on the metal and μ_s is the coefficient of maximum static friction.

Each revolution of the crank will raise the end of the board 6.3 mm. The mass of the metal is 0.10 kg. The length of the board is 60 cm.

最大静摩擦力可表示为$f_{max} = \mu_s N$，其中N为金属块受到的正向力，μ_s为最大静摩擦系数。

曲柄每转动一圈，斜面一端的高度会增 6.3 mm。金属块质量为 0.10 kg。平板长度为 60cm。



Question 27

Objective: Determine the refractive index of a glass. **Keep 2 significant figures for your answer**.

In this virtual lab, you can
1. Click 'Laser On' to turn on the laser.
2. Use the mouse to drag the laser to change its position.
3. Read the position of the laser from screen.
4. Click 'Reset' button to reset the setup.

实验目的: 求玻璃块的折射率。**答案取 2 位有效数字**。

在此虚拟实验室中， 你可以
1. 点击'Laser On' 开启镭射。
2. 使用滑鼠拖曳镭射改变其位置。
3. 从荧幕上读取镭射位置。
4. 点击 'Reset'恢复初始设定。

According to Snell's law,

$$n_i \sin\theta_i = n_r \sin\theta_r$$

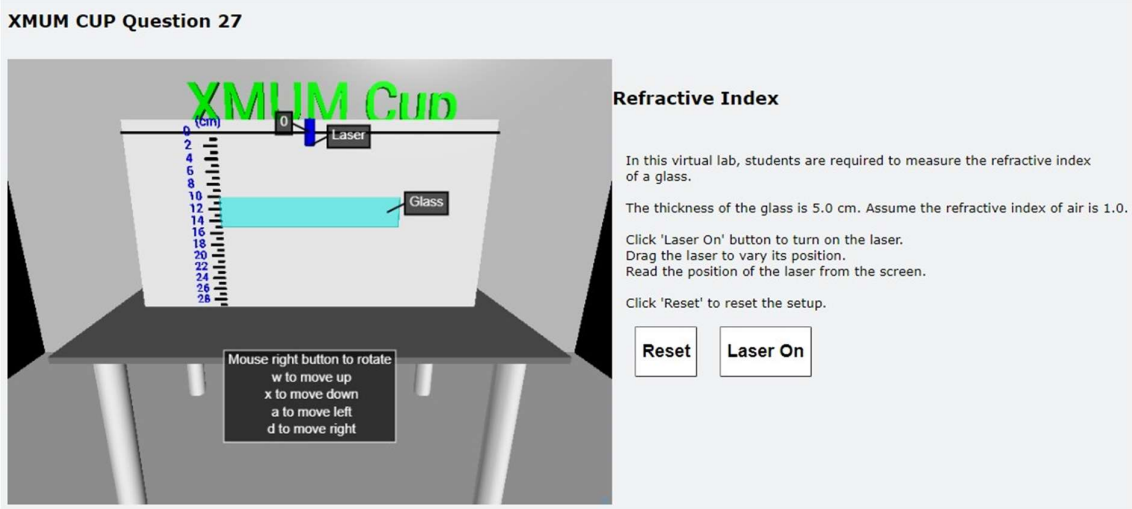
Where θ_i and θ_r denote the angles of incidence and refraction. n_i and n_r are the refractive indices of the respective medium.
In the experiment, the thickness of the glass is 5.0 cm. Assume the refractive index of air is 1.0.

根据 Snell's law,

$$n_i \sin\theta_i = n_r \sin\theta_r$$

其中θ_i 和 θ_r 分别表示入射与折射角。 n_i 和 n_r 为各别介质的折射率。

实验中的玻璃块厚度为 5.0 cm，取空气的折射率为 1.0。



Question 28

Objective: Determine the work function of the metal used in the photoelectric effect experiment. **Give the answer in unit of eV to 2 significant figures.**

In this virtual lab, you can
1. Choose 4 different light sources by clicking the colour button.
2. Adjust the voltage by dragging the sliding bar.
3. Read the current and voltage from the graph.

实验目的：求光电效应实验中，金属的功函数。**答案以 eV 为单位取 2 位有效数字。**

在此虚拟实验室中， 你可以
1. 用滑鼠点击颜色按钮以选择 4 种不同光源。
2. 拖曳滑块以调整光电管两端电压。
3. 从图中读取电流与电压的值。

Einstein's equation gives

$$E_k = \alpha f - W$$

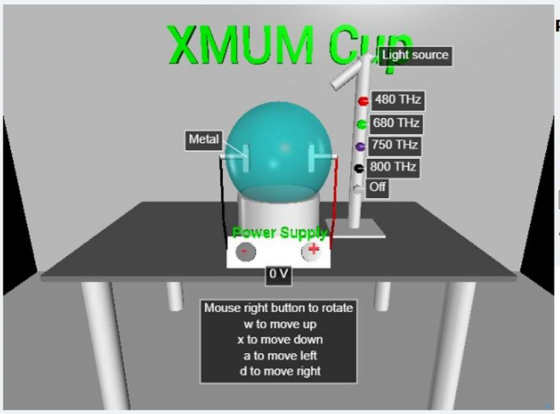
Where E_k is the kinetic energy of photoelectron, f is the frequency of photon, W is the work function of the metal, α is an unknown constant.

由爱因斯坦方程知

$$E_k = \alpha f - W$$

其中E_k为光电子的动能，f为光子的频率，W为金属的功函数，α为未知常数。

XMUM CUP Question 28



Photoelectric Experiment

In this virtual lab, students are required to obtain the work function of the metal used in the photoelectric effec experiment.

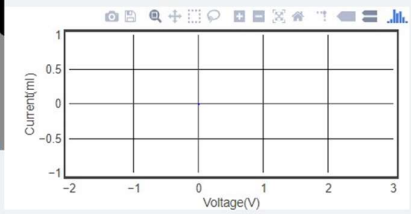
Four different frequecies of light sources are available.
480Thz, 680THz, 750THz, 800THz

Click the color button on the light source to turn on the light.
Drag the sliding bar to to adjust voltage.
The current will be measured and plotted in the graph automatically.
Click "Reset" to reset the setup.

Reset

Adjust the voltage here.

0.00 V



********** End of Paper **********

****试卷结束****